Macro IV

Problem Set 4

due Thursday, April 21^{st} at 9.00 a.m.

1. Consider a generalized spillover model a la Lucas (1988). The production function of each firm is $Y_i = K_i^{\alpha}(A_i L_i)^{(1-\alpha)}$, where K_i is firm-specific capital, L_i is firm-specific labor, and A_i is the spillover effect on labor productivity, $\alpha \in (0,1)$. Assume also that $A_i = \frac{K}{L}$ where K and L are aggregate capital and labor. Assume CES preferences and no population growth, hence the Euler equation for the household problem is $\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho)$.

   (a) (0.5 points) Derive private marginal product of capital $\frac{\partial Y_i}{\partial K_i}$. Use market clearing conditions to derive Euler equation in competitive equilibrium. Is there endogenous growth? Is there a scale effect (effect of L on the endogenous growth rate)?

   (b) (1 point) Now assume that there is a benevolent central planner that maximizes the utility of the household subject to the constraint $\dot{K} = F(K, AL) - cL - \delta K$. Derive the Euler equation of the central planner.

   (c) (0.5 points) Compare your answers in (a) and (b). Which economy will have faster growth - centralized or decentralized? Why?

2. Consider the Romer model. The stock of patents A_t increases according to the following law: $\dot{A}_t = \phi A_t^{\gamma} L_{A,t}^{\eta}$, where $L_{A,t}^{\gamma}$ is the number of workers employed in R&D, and $\gamma > 0$, $\eta \in (0,1)$. The final good is produced according to $Y_t = K_t^{\alpha}(A_t L_{Y,t})^{1-\alpha}$. Assume that λ (the fraction of workers employed in final goods sector) does not change over time. Finally, assume that L_t grows at a constant rate n.

   (a) (0.5 points) To which value do we have to set parameter γ, if we want to assume that there are no externalities of old patents on the creation of new ones?

   (b) (0.5 points) Assume the growth rate of A_t is constant in the steady sate. Show that in the steady state the growth rate of A_t is given by $g_A = \frac{\eta}{1-\gamma} n$.

   (c) (0.5 points) Assume $k_t = \frac{K_t}{L_t}$ and $y_t = \frac{Y_t}{L_t}$ grow at the same constant rate in the steady state. Find the steady-state growth rate of k_t and y_t as a function of g_A.

   (d) (0.5 points) Consider the following statement: "Ideally, we should set $\lambda = 0$ in order to maximize growth in this economy". Is this statement correct? Say why, or why not (At most two sentences).

   (e) (1 point) Is the speed of innovation g_A optimal in this economy? Explain in at most three sentences.