Macro IV

Problem Set 3

due Thursday, April 14^{th} at 9.00 a.m.

1. Consider a centralized version of the Ramsey model. Now instead of a firm and a household we only have a central planner with the direct access to technology $F(K_t, A_t L_t)$. The central planner is maximizing $U = \int_0^\infty u[c(t)]e^{nt}e^{-\rho t}dt$ subject to $\dot{K} = F(K, AL) - cL - \delta K$; $K(0) = K_0 > 0$; and $K(t) \geq 0$ for any t.
   (a) (0.5 points) Derive a law of motion for $k(t) = \frac{K(t)}{L(t)}$ (in other words, derive an expression for $\dot{k}$).
   (b) (1 point) Assume CES preferences from now on. Write a Hamiltonian for the central planner problem using your result in (a). Derive first order conditions. Use them to derive Euler equation.
   (c) (0.5 points) Euler equation, transversality condition and the law of motion of k now describe the model completely. Compare this set of equations to the set of equations that characterized the competitive equilibrium in the decentralized version of the Ramsey model we saw in class. Comment on the Pareto optimality of the decentralized version.

2. Consider again a centralized version of the Ramsey model. We only have a central planner with the direct access to technology $F(K_t, A_t L_t)$. Now there is an amount of government expenditures $gA(t)L(t)$ that need to be financed, where g is a constant parameter. The central planner is maximizing $U = \int_0^\infty u[c(t)]e^{nt}e^{-\rho t}dt$ subject to $\dot{K} = F(K, AL) - cL - gAL - \delta K$; $K(0) = K_0 > 0$; and $K(t) \geq 0$ for any t.
   (a) (1 point) Derive a law of motion for $k(t) = \frac{K(t)}{L(t)}$ (in other words, derive an expression for $\dot{k}$). Assume CES preferences from now on. Write a Hamiltonian for the central planner problem. Derive first order conditions and Euler equation.
   (b) (1 point) Euler equation, transversality condition and the law of motion of k now describe the model completely. Rewrite them in terms of $\hat{k} = \frac{k}{A}$ and $\hat{c} = \frac{c}{A}$. Apply steady-state conditions ($\dot{\hat{c}} = 0$ and $\dot{\hat{k}} = 0$) and draw a phase diagram in $\hat{c}$ and $\hat{k}$.
   (c) (1 point) Use phase diagram to find the effect of an increase in g on the steady-state allocations of $\hat{c}$ and $\hat{k}$.