Macro IV
Problem Set 2
due Thursday, April 7^{th} at 9.00 a.m.

Problem 1. (2 points). Consider a Solow-Swan model *without technological progress*. Capital depreciates at a rate $\delta \in (0.1)$. The production function is $Y_t = K_t^\alpha (L_t)^{1-\alpha}$. Only a constant share l of population is working, so that the number of workers is $L_t = lN_t$ and the number of unemployed is $U_t = (1-l)N_t$. There is also a government that taxes away a part of workers' income τY_t and redistributes it evenly among the unemployed. The population is growing at a rate $\frac{\dot{N}_t}{N_t} = n$. The workers save the constant share s of their income $(1-\tau)Y_t$. Unemployed do not save.

a) Derive the law of movement for capital K_t and capital per capita $k_t = \frac{K_t}{N_t}$.

b) Find the steady state values of income per capita for workers (i.e. $(1-\tau)Y_t/L_t$) and unemployed ($\tau Y_t/U_t$).

c) If a person had a choice to become employed or unemployed, under what conditions would she/he choose unemployment?

d) Under which conditions would unemployed vote for the increase in tax rate τ?

e) What is the impact of the permanent decrease in l on the steady-state level of worker's income per capita? Explain the result.

Problem 2. (1 point). Consider a modified version of Solow-Swan model *without technological progress*. Aggregate production function is $F(K, L, Z) = K^\alpha L^\beta Z^{1-\alpha-\beta}$, where Z is land available in fixed supply. Capital depreciation rate is δ, and a fixed share of income s is saved.

a) Suppose there is no population growth. Find steady-state capital-labor ratio and output level.

b) Now suppose population grows at rate n. What happens to the capital-labor ratio and output level as $t \to \infty$? What happens to returns to land and the wage rate as $t \to \infty$?

Problem 3. (1 point). Consider a Solow-Swan model *with labor-augmenting technological progress* we discussed in class. Assume that until the moment $t = 0$ the economy was in the steady state (balanced growth path) associated with the parameters x, s, δ, n of the model. Analyze the transition to the new steady state (both in the short and in the long run) if the following permanent changes happen to one of the parameters, keeping the rest constant. Use graphs to depict the evolution of $\hat{k}_t, log(k_t)$ and $log(y_t)$.

a) Permanent increase in the depreciation rate δ.

b) Permanent decrease in the level of technology A_t (but not in it's growth rate x).

c) Permanent increase in the rate of technological progress x.

Problem 4. (0.5 points). Consider a Solow-Swan model *with labor-augmenting technological progress* we discussed in class. Assume the production function is Cobb-Douglas ($Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$) and that until the moment $t = 0$ the economy was in the steady state (on the balanced growth path) associated with the parameters x, s, δ, n, α of the model. Assume in the moment $t = 0$ the level of labor-augmenting technology jumps upward by 10% (the rate of technological progress x stays constant).

a) How much is the new steady state (balanced growth path) higher than the old one in terms of output per worker?.

b) Assume the economy has finished the transition from the old steady state to the new one. If you do the growth accounting exercise (using the primary method), how much of the increase in the steady-state output per worker is due to the increase in capital per worker, and how much is due to the increase in technology?

c) In what sense are the results of the growth accounting exercise in part b) wrong? And in what sense are they correct?

Problem 5. (0.5 points). Consider a Ramsey model where representative household is maximizing $U = \int_0^{\infty} u[c(t)]e^{nt}e^{-\rho t}dt$. Assume $u[c(t)] = log[c(t)]$ and $\dot{a} = w + ra - (1+\tau)c - na$, where τ is the tax on consumption.

a) Write a Hamiltonian for the household problem.

b) Take first order conditions for a maximum of U.

c) Derive Euler equation from the first order conditions.